Instrumental Variation - Part II

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AREC 829
Policy Design and Causal Inference for Social Science

Latent Index Models

Generalized Roy Model

- Assume that Y_0 and Y_1 are potential earnings from dropping out after high school and from attending college, respectively.
- The school choice decision (high school vs college) is D.
- **Potential outcome equations** are defined for $j = 0, 1$ as:

$$Y_j = \mu_j(x) + U_j, \tag{1}$$

  where X is a set of observables and $\mu_j(x) \equiv E[Y_j|X = x]$.
- The innocuous normalization $E[U_j|X = x] = 0$ is considered, and the variables U_j need not be independent of X (in most applications, additional restrictions are imposed).
- I will write an equation for the 'treatment' choice that depends on potential outcomes.

Generalized Roy Model

- Choices are made by agents under **imperfect information** about costs and outcomes that will occur, $\mathcal{I}$ being the information set at time of decision. The **expected benefit** at time of decision is:

$$I_D \equiv E[Y_1 - Y_0 - C|\mathcal{I}],$$

  where, for example, C represents tuition fees and $D = \mathbb{1}(I_D > 0)$.

- Policies typically operate by changing (Y_0, Y_1) or C (e.g., reducing tuition or commuting costs, or taxing future earnings).
- Consider $X \subset \mathbf{Z} \equiv [X, Z]$ (the notation here embeds an **exclusion restriction**) and:

$$C = \mu_C(\mathbf{Z}) + U_C,$$

  where $\mathbf{Z}$ may consists of **multiple excluded instruments** and $\mu_C(\mathbf{Z}) \equiv E[C|\mathbf{Z} = \mathbf{z}]$.

Normalization of the Choice Equation

- We can re-arrange terms to write:

$$I_D = \mu_D(\mathbf{Z}) - V, \qquad \underbrace{D = \mathbb{1}(\mu_D(\mathbf{Z}) > V)}_{decision\ \ to\ \ enroll},$$

the elements of the index model being defined as follows:

$$\begin{aligned} \mu_D(\mathbf{Z}) &\equiv E[\mu_1(X) - \mu_0(X) - \mu_C(\mathbf{Z})|\mathcal{I}], \\ V &\equiv -E[U_1 - U_0 - U_C|\mathcal{I}]. \end{aligned}$$

- Define the following **propensity score** with respect to $\mathbf{Z}$:

$$P[\mathbf{Z}] \equiv P[\mu_D(\mathbf{Z}) > V] = F_V[\mu_D(\mathbf{Z})].$$

- This implies that the **schooling equation** can be written as:

$$D = \mathbb{1}(F_V[\mu_D(\mathbf{Z})] > F_V[V]) = \mathbb{1}(P[\mathbf{Z}] > U_D), \tag{2}$$

where U_D is a random variable uniformly distributed over $[0, 1]$.

Assumptions

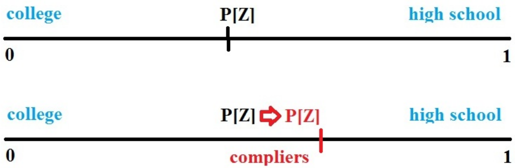
- The model (1) and (2) is completed by assuming that (a) $(U_0, U_1, V) \perp \mathbf{Z} | X$ or $(U_0, U_1, U_D) \perp \mathbf{Z} | X$; (b) $\mu_D(\mathbf{Z})$ is non-degenerate given X; (c) V has a **continuous** distribution.
- Both the control X and exogenous components of $\mathbf{Z}$ may be continuous, discrete and ordered, categorical, or binary.
- The equations above define a **latent index model of choice and treatment effects**.
- $E[Y_1 - Y_0 | I_D = 0]$ is the (average) **effect for units at the margin of indifference** for enrolment in the 'programme' (i.e., higher education).
- The **marginal** agents here are those with $\{I_D = 0\} \equiv \{U_D = P[\mathbf{Z}]\}$.
- The instrument, $\mathbf{Z}$, may be a vector of discrete or continuous random variables (to fix ideas, instruments here are tuition fees).

An Equivalence Result

- The assumptions underlying the (continuous latent index) Roy model developed are **equivalent** to those in the LATE setting.
- The treatment choice underlying LATE coincides with equation (2):

$$D = \mathbb{1}(P[\mathbf{Z}] > U_D).$$

- **Additive separability** between $P[\mathbf{Z}]$ and U_D in the index restriction is required for the result to hold (U_D is **distaste for college** in this metric, or the unobserved resistance to receive treatment).
- It plays a key role in the IV methodology: additive separability combined with **only one** latent (i.e., unobservable) index imply and are implied by the monotonicity assumption.



Some Facts About Latent Index Restrictions

- The latent index channels all unobserved determinants of treatment status **through one single variable** (college distaste) and the variation of treatment effects by this latent variable captures all of the unobserved heterogeneity related selection bias.
- Substantive restrictions on choice behavior.
- Under this model, **treatment effect heterogeneity occurs only along two dimensions**.
  - The observed probability of treatment (the propensity score): $P[\mathbf{Z}]$.
  - The latent variable for unobserved distaste: U_D.
  - Conditional on $P[\mathbf{Z}]$ and U_D, treatment effect $Y_1 - Y_0$ must be independent of D because of equation (2).
- Intuitively, **treatment effect parameters become a bivariate function of the propensity score and the latent variable**.
- You may recall I discussed this already in the previous set of slides.

Some Facts About Monotonicity

- **If the instrument is exactly the policy of interest, then LATE is the relevant parameter to estimate**. This means that for some questions it is the relevant parameter, but for others it is not (this is an external validity problem).
- Condition on X throughout. Policy: reduction of tuition fees.
- Monotonicity states that fixing the instrument Z at two values z_1 and z_2 moves choices across agents **in the same direction**.
- There is either $D(z_1) \leq D(z_2)$ or $D(z_1) \geq D(z_2)$ **for all agents**.
- The condition does not require that the direction of inequalities is the same over the support of Z: for example, there might be $D(z_1) \leq D(z_2)$ and $D(z_3) \geq D(z_4)$.
- If a proposed policy operates to change Z at different values (e.g. z_3 and z_4), $LATE(z_1, z_2)$ does **not** identify $LATE(z_3, z_4)$.

Some Facts About Monotonicity

- Under the conditions stated the return to participation for agents changing the treatment status as a result of the switch from $Z = z_1$ to $Z = z_2$ (compliers) is:

$$LATE(z_1, z_2) = E[Y_1 - Y_0 | D(z_2) = 1, D(z_1) = 0].$$

- Z enters the model only through its effect on the score $P[\mathbf{Z}]$, which is called **index sufficiency**. As a consequence of this we have:

$$LATE(z_1, z_2) = E[Y_1 - Y_0 | P(z_1) \leq U_D \leq P(z_2)].$$

- **Any configuration of instruments that yields values p_1 and p_2 of $P[Z]$ defines the same LATE**.
- LATE does not identify $E[Y_1 - Y_0 | I_D = 0]$ in general. It identifies the **returns induced to attending college as a result of an instrument (or policy) change**.

Local Instrumental Variables

Marginal Treatment Effects

- The conditioning on X if implicit throughout.
- The orthogonality properties of Z imply:

$$
\begin{aligned}
E[Y|P(Z)=p] &= E[Y_0 + \mathbb{1}(p > U_D)(Y_1 - Y_0)|P(Z)=p], \\
&= E[Y_0 + \mathbb{1}(p > U_D)(Y_1 - Y_0)], \\
&= E[Y_0] + E[Y_1 - Y_0|p > U_D]P[p > U_D], \\
&= E[Y_0] + \underbrace{E[Y_1 - Y_0|p > U_D]p}_{S(p)}, \qquad (3) \\
&= E[Y_0] + \int_0^p \frac{\partial S(\eta)}{\partial \eta} d\eta. \qquad (4)
\end{aligned}
$$

- The last equality follows from the theorem for the derivative of an integral.
- Note that $S(p) = E[\mathbb{1}(p > U_D)(Y_1 - Y_0)]$ can be interpreted as the (average) **gross gain in the population** (no gain if $D = 0$).

Marginal Treatment Effects

- Using the properties of random variables involved:

$$\frac{\partial E[Y|P(Z)=p]}{\partial p} = \frac{\partial S(p)}{\partial p} = E[Y_1 - Y_0|U_D = p].$$

- The quantity (the conditioning on X is now explicit):

$$MTE(u,x) = E[Y_1 - Y_0|U_D = u, X = x],$$

  is called **marginal treatment effect (MTE)** at $U_D = u$ and $X = x$.

- It is the return for the agent on a **margin of indifference** between participation (college) and non-participation (high school).
- Since $U_D \in [0,1]$, it describes how returns vary across quantiles of the unobserved component U_D. Each quantile identifies who is induced to go to college by a marginal change in $P(Z)$.
- The sample analogue of $MTE(u,x)$ defines the **local instrumental variable (LIV)** estimator.

- If $p_2 > p_1$ we have $\mathbb{1}(p_1 > U_D) = 1 \Rightarrow \mathbb{1}(p_2 > U_D) = 1$ and $\mathbb{1}(p_2 > U_D) = 0 \Rightarrow \mathbb{1}(p_1 > U_D) = 0$. If follows that:

$$\begin{aligned} S(p_2) - S(p_1) &= E[Y_1 - Y_0 | p_1 \leq U_D \leq p_2](p_2 - p_1), \\ &= \int_{p_1}^{p_2} MTE(\eta) d\eta. \end{aligned}$$

- This implies that LATE is the average MTE with weights $1/(p_2 - p_1)$ in the range $[p_1, p_2]$:

$$LATE(p_1, p_2) = \frac{S(p_2) - S(p_1)}{p_2 - p_1} = \frac{\int_{p_1}^{p_2} MTE(\eta) d\eta}{p_2 - p_1}.$$

- LIV is 'local' in the following sense:

$$\lim_{p_2 \to p_1} LATE(p_1, p_2) = \lim_{p_2 \to p_1} \frac{S(p_2) - S(p_1)}{p_2 - p_1} = MTE(p_1).$$

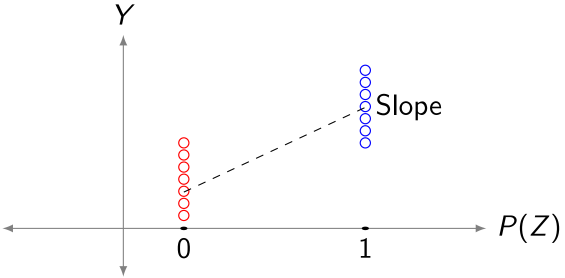
Relationship between LATE and LIV

- It turns out that **all traditional treatment effect parameters can be expressed as weighted averages of the MTE integrating with respect to agents at different margins**.
- For example, integrating over the full support of U_D one gets $ATE = \int_0^1 MTE(\eta) d\eta$ (intuitively, we are integrating across all individuals).
- The relationship with LATE is also evident through the LIV interpretation.
- More in general, many treatment parameters can be written as:

$$\underbrace{\Delta_j(x)}_{parameter} = \int_0^1 MTE(u,x) \underbrace{h_j(u,x)}_{weights} du, \tag{5}$$

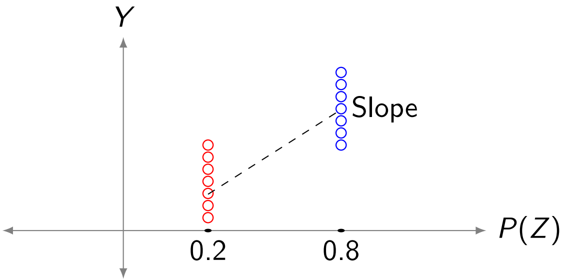
with weights that are parameter-specific.

Binary Instrument Z: Full Compliance



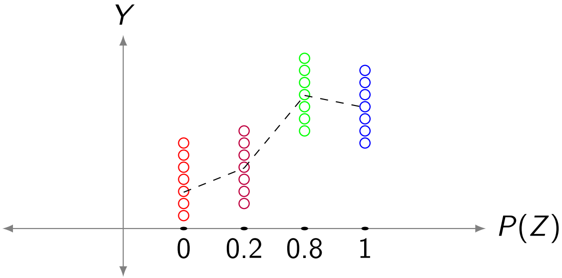
$$
\begin{aligned}
Slope &= \frac{E\left(Y|P(Z=1)\right) - E\left(Y|P(Z=0)\right)}{P(Z=1) - P(Z=0)}, \\
&= \frac{E\left(Y|Z=1\right) - E\left(Y|Z=0\right)}{1-0}, \\
&= E\left(Y|D=1\right) - E\left(Y|D=0\right) = ATE.
\end{aligned}
$$

Binary Instrument Z: Imperfect Compliance



$$
\begin{aligned}
Slope &= \frac{E\left(Y|P(Z=1)\right) - E\left(Y|P(Z=0)\right)}{P(Z=1) - P(Z=0)}, \\
&= \frac{E\left(Y|Z=1\right) - E\left(Y|Z=0\right)}{0.8 - 0.2}, \\
&= \frac{E\left(Y|Z=1\right) - E\left(Y|Z=0\right)}{E\left(D|Z=1\right) - E\left(D|Z=0\right)} = LATE.
\end{aligned}
$$

Categorical and Continuous Instruments



$$\begin{aligned} Slope(z'', z') &= \frac{E(Y|Z = z'') - E(Y|Z = z')}{E(D|Z = z'') - E(D|Z = z')} = LATE(z'', z'), \\ MTE(z') &= \lim_{z'' \to z'} LATE(z'', z'). \end{aligned}$$

Estimation

Example: Returns to Education

- Sample of white males aged 28 – 34 in 1991 from **NLSY data**.
- Outcome variable (Y): log wages in 1991.
- Treatment variable (D): ever enrolled in college by 1991.
- Conditioning variables (X): cognitive ability (AFQT), maternal education, number of siblings, years of experience in 1991, cohort dummies, and a bunch of area level controls.
- Multiple Instruments (Z): (1) presence of a four year public college at age 14, (2) log average earnings when 17 (opportunity cost), (3) average unemployment rate in state when 17, (4) local tuition in four year public college at age 17.

LIV Estimation

- Under the standard IV assumptions, $MTE(u, x)$ is identified by:

$$\frac{\partial E[Y|P(\mathbf{Z}) = u, X = x]}{\partial u} = E[Y_1 - Y_0|U_D = u, X = x].$$

- **LIV estimates directly this difference**.
- Identification requires that the excluded instruments in $\mathbf{Z}$ vary sufficiently (given X) to make the treatment assignment probability $P[\mathbf{Z}]$ vary anywhere in $[0, 1]$.
- Instruments rarely provide this variation in empirical work.
- Aggregating multiple instruments into the scalar index $P[\mathbf{Z}]$ enlarges the range of values over which one can identify MTE (in comparison to using each instrument one at a time).
- A frequently made assumption to ease estimation restricts the **shape** of $MTE(u, x)$ not to vary with X.

LIV Estimation under Additive Separability

- Specifically, **additive separability** in X and U_D for $j = 0, 1$ implies the following form for the two **marginal treatment response (MTR)** functions:

$$\underbrace{E[Y_j|U_D = u, X = x]}_{m_j(u,x)} = \mu_j(x) + \underbrace{E[U_j|U_D = u]}_{independent\ \ of\ \ X}. \tag{6}$$

- This is also implied by $(X, Z) \perp (U_0, U_1, U_D)$, which is stronger than the conditional independence assumption.
- Let $\mu_j(X) = \tau_j X$ for $j = 0, 1$. Using (3) we have:

$$\begin{aligned} E[Y|P(\mathbf{Z}) = u, X = x] &= \tau_0 x + (\tau_1 - \tau_0)xu \\ &+ E[U_1 - U_0|u > U_D]u, \end{aligned} \tag{7}$$

and by taking the derivative, under additive separability we have:

$$MTE(u, x) = (\tau_1 - \tau_0)x + \underbrace{E[U_1 - U_0|U_D = u]}_{independent\ \ of\ \ X}.$$

LIV Estimation under Additive Separability

- Note that (7) is nonlinear in u: with a binary instrument the empirical analogues of $E[Y|P(\mathbf{Z}) = u, X = x]$ are only two points and LIV is **not identified**.
- Intuition: MTE can be linear at worst, and to identify a linear derivative one needs at least three points.
- Additive separability buys identification over the unconditional support of $P[\mathbf{Z}]$, rather than the support of $P[\mathbf{Z}]$ conditional on X.
- This paves the way for **semi-parametric** estimation.
  - Estimation requires a pre-estimated propensity score $P(\mathbf{Z})$.
  - Estimate τ_0 and τ_1 by partialing out $P[\mathbf{Z}]$ in (7).
  - Estimate $E[U_1 - U_0|U_D = u]$ as first derivative computed from a local polynomial regression using $Y - \hat{\tau}_0 X - (\hat{\tau}_1 - \hat{\tau}_0)P(\mathbf{Z})X$.

Estimation under Discrete Variation

- While LIV works directly with $E[Y|P(\mathbf{Z}) = u, X = x]$ in (7) to obtain the expectation of the difference $E[Y_1 - Y_0|U_D = u, X = x]$, an alternative approach is to estimate the difference of expectations by considering **one expectation at the time**.
- Consider for example (the conditioning on X is left implicit):

$$\begin{aligned} E[Y|P(\mathbf{Z}) = u, D = 1] &= \mu_1 + E[U_1|P(\mathbf{Z}) = u, U_D < u], \\ &= \mu_1 + E[U_1|U_D < u]. \end{aligned}$$

- Recall that:

$$E[U_1|U_D = u] = \frac{\partial E[U_1|U_D < u]u}{\partial u},$$

which by taking the derivative of the product implies:

$$\begin{aligned} E[U_1|U_D = u] &= u\frac{\partial E[U_1|U_D < u]}{\partial u} + E[U_1|U_D < u], \\ &= u\frac{\partial E[Y|P(\mathbf{Z}) = u, D = 1]}{\partial u} + E[U_1|U_D < u]. \end{aligned}$$

Estimation under Discrete Variation

- It follows that:

$$\underbrace{E[U_1|U_D = u] + \mu_1}_{MTR \;\; for \;\; Y_1} = u\frac{\partial E[Y|P(\mathbf{Z}) = u, D = 1]}{\partial u} + E[Y|P(\mathbf{Z}) = u, D = 1]. \tag{8}$$

- Similar calculations yield:

$$\begin{aligned} E[Y|P(\mathbf{Z}) = u, D = 0] &= \mu_0 + E[U_0|U_D \geq u], \\ E[U_0|U_D = u] &= -(1-u)\frac{\partial E[U_0|U_D \geq u]}{\partial u} + E[U_0|U_D \geq u], \\ \underbrace{E[U_0|U_D = u] + \mu_0}_{MTR \;\; for \;\; Y_0} &= -(1-u)\frac{\partial E[Y|P(\mathbf{Z}) = u, D = 0]}{\partial u} \\ &\quad + E[Y|P(\mathbf{Z}) = u, D = 0]. \end{aligned} \tag{9}$$

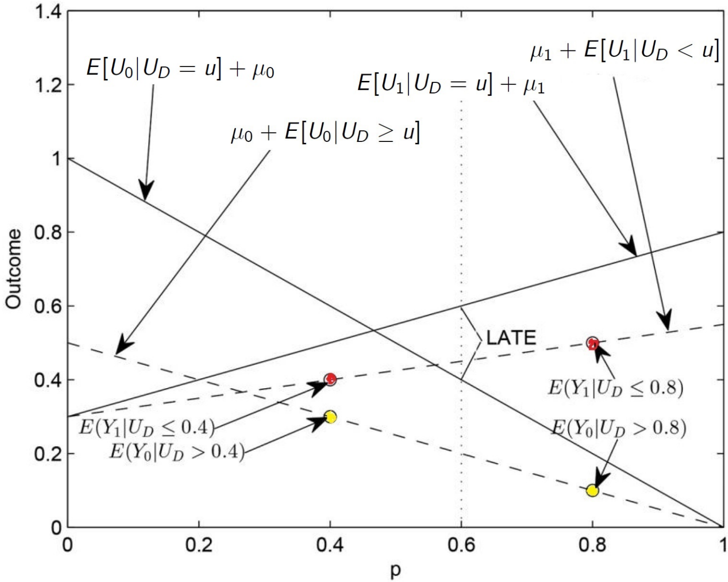
- The difference between (8) and (9) identifies:

$$MTE(u) = \mu_1 - \mu_0 + E[U_1 - U_0|U_D = u].$$

Estimation under Discrete Variation

- **Semiparametric estimation** is again possible via local polynomial regressions.
- The latter approach is convenient with **discrete instruments** and without additive separability.
- To see this, assume that $P(\mathbf{Z})$ takes on only two values (given X) because of variation induced by a binary instrument.
- Consider the case of $MTE(u)$ linear in u.
- This implies that $E[Y|P(\mathbf{Z}) = u]$ must be quadratic in u, and that LIV cannot identify the linear MTE model.
- However, since $E[U_1|U_D = u]$ and $E[U_0|U_D = u]$ must be linear in u, a binary instrument is sufficient to identify the linear MTE model from (8) and (9).

Estimation under Discrete Variation



Ex Ante Evaluation

Instrumental Variation and Policy Effects

- The aim is to **forecast the effects of a policy change**.
- Consider a policy that introduces a **marginal** expansion in treatment participation by increasing $P(\mathbf{Z})$.
- To fix ideas, consider the increase in college attendance resulting from a marginal reduction in tuition fees.
- Agents induced to treatment (college) by a particular variation of the instruments $\mathbf{Z}$ in the data need not be the same agents induced to treatment by the policy change considered.
- Can the effects of a marginal expansion be identified without requiring that the variation considered by the policy corresponds exactly to the variation in the instruments $\mathbf{Z}$?
- This setup echoes the idea of forecasting the effects of an intervention from the current to a different environment, or of an intervention never historically implemented (**policy counterfactuals**).

Instrumental Variation and Policy Effects

- Let b ("before") represent the baseline policy regime, and a ("after") the alternative policy regime after the marginal change.
- With obvious notation, (Y_0^b, Y_1^b) and (Y_0^a, Y_1^a) are the potential outcomes under the two policy regimes.
- Some notion of 'stability' is required.
  - A tuition policy for college enrolment is - in the **absence of general equilibrium effects** - an example of one commonly invoked form of **policy invariance**, where potential earnings are unaffected ($Y_0^b = Y_0^a$ and $Y_1^b = Y_1^a$), costs vary under the two scenarios and changes in costs affect incentives to enroll.
  - Taste is unchanged: $U_D^b = U_D^a$.
  - Stability of distributions is enough in general, possibly conditional on observables.

- In this setting, if $Y^a = D^a Y_1 + (1 - D^a) Y_0$ and $Y^b = D^b Y_1 + (1 - D^b) Y_0$ are the observed outcomes, and $E(Y^a - Y^b)$ is the **gross benefit**.
- Unless $D^a = 1$ and $D^b = 0$, the parameter $E(Y^a - Y^b)$ will be different in general from the standard average effect.
- The **policy relevant treatment effect** (PRTE) is the normalized (per-person) benefit:

$$\frac{E(Y^a - Y^b)}{E(D^a - D^b)}.$$

- The notion of 'stability' implies that LATE and MTE are **invariant** to the set of instruments yielding the same value of the score $P[Z]$:

$$\begin{aligned} LATE(z_1, z_2) &= E[Y_1 - Y_0 | P(z_1) \leq U_D \leq P(z_2)], \\ MTE(u) &= E[Y_1 - Y_0 | U_D = u]. \end{aligned}$$

Identification in a Nutshell

- Denote $P[\mathbf{Z}]$ by $\mathbf{P}$. Using (4) we have (conditional on X):

$$
\begin{aligned}
E[Y] &= \int_0^1 E[Y|\mathbf{P}=p] f_{\mathbf{P}}(p)dp, \\
&= E[Y_0] + \int_0^1 \int_0^p MTE(\eta) f_{\mathbf{P}}(p) d\eta dp, \\
&= E[Y_0] + \int_0^1 MTE(\eta) \left[\int_\eta^1 f_{\mathbf{P}}(p)dp \right] d\eta, \\
&= E[Y_0] + \int_0^1 MTE(\eta)[1 - F_{\mathbf{P}}(\eta)] d\eta, \\
E(Y^a - Y^b) &= \int_0^1 \underbrace{MTE(\eta)}_{invariant} [F_{\mathbf{P}^b}(\eta) - F_{\mathbf{P}^a}(\eta)] d\eta.
\end{aligned}
$$

- The policy change enters **only** through the distribution of $P[\mathbf{Z}]$.

Identification in a Nutshell

- The PRTE is defined for a change from a baseline policy to a fixed alternative.
- Identifying it in any sample can be challenging as $P(\mathbf{Z})$ will have to vary over the full unit interval.
- A less empirically demanding quantity to estimate is a **marginal PRTE** (MPRTE) corresponding to the effect of a small change from a baseline policy.
- Identification is triggered by the perturbation in $P(\mathbf{Z})$.
- Alternative policies can be indexed to a scalar α regulating the nature of the 'perturbation', with $\alpha = 0$ denoting the baseline policy.
- To fix ideas, consider a policy increasing the probability of college enrolment by an amount α, so that $P^a = P^b + \alpha$.
- It follows that $F_{\mathbf{P}^a}(\eta) = F_{\mathbf{P}^b}(\eta - \alpha)$.

Nuts and Bolts (intuition)

- Under fair regularity conditions there is (conditioning on X):

$$
\begin{aligned}
E(Y^a - Y^b) &= \int_0^1 MTE(\eta)[F_{\mathbf{P}^b}(\eta) - F_{\mathbf{P}^a}(\eta)]d\eta, \\
&= \alpha \int_0^1 \underbrace{MTE(\eta)}_{\textit{invariant}} \left[\frac{F_{\mathbf{P}^b}(\eta) - F_{\mathbf{P}^b}(\eta - \alpha)}{\alpha} \right] d\eta, \\
\lim_{\alpha \to 0} E(Y^a - Y^b) &\simeq \alpha \int_0^1 MTE(\eta) f_{\mathbf{P}^b}(\eta) d\eta.
\end{aligned}
$$

- There are some technical issues arising from the approximation of the indifference set $P[\mathbf{Z}] = U_D$ through limit operations.
- Using this idea it is possible to show that:

$$MPRTE = \int_0^1 MTE(\eta) h_{MPRTE}(\eta) d\eta.$$

which represents a weighted average derivative with weights determined by the marginal policy of interest.

Example: Returns to Education

TABLE 5—RETURNS TO A YEAR OF COLLEGE

Model		Normal	Semiparametric
$ATE = E(\beta)$		0.0670 (0.0378)	Not identified
$TT = E(\beta \mid S = 1)$		0.1433 (0.0346)	Not identified
$TUT = E(\beta \mid S = 0)$		−0.0066 (0.0707)	Not identified
MPRTE			
Policy perturbation	Metric		
$Z_\alpha^k = Z^k + \alpha$	$\lvert \mathbf{Z}\gamma - V \rvert < e$	0.0662 (0.0373)	0.0802 (0.0424)
$P_\alpha = P + \alpha$	$\lvert P - U \rvert < e$	0.0637 (0.0379)	0.0865 (0.0455)
$P_\alpha = (1 + \alpha)P$	$\lvert \frac{P}{U} - 1 \rvert < e$	0.0363 (0.0569)	0.0148 (0.0589)
Linear IV (Using $P(\mathbf{Z})$ as the instrument)		0.0951 (0.0386)	
OLS		0.0836 (0.0068)	

Notes: This table presents estimates of various returns to college, for the semiparametric and the normal selection models: average treatment effect (ATE), treatment on the treated (TT), treatment on the untreated (TUT), and different versions of the marginal policy relevant treatment effect (MPRTE). The linear IV estimate uses P as the instrument. Standard errors are bootstrapped (250 replications). See online Appendix Table A-1 for the exact definitions of the weights. See Table 1 for the weights for MPRTE. For more discussion of MPRTE, see Carneiro, Heckman, and Vytlacil (2010).

ExtrapoLATE

Intuition

- How to use IV to learn about treatment effect parameters **other than LATE**? Or for a group **different from compliers**?
- I touched upon the idea in (5) that LATE (and many treatment effect parameters) can be written as weighted averages of the MTE.
- Using (6), a similar idea extends to MTR functions:

$$\underbrace{\Delta_j}_{parameter} = E\left[\int_0^1 m_1(u,x)\underbrace{\omega_1(u,x)}_{weight}\,du\right] + E\left[\int_0^1 m_0(u,x)\underbrace{\omega_0(u,x)}_{weight}\,du\right].$$

- Weights in this average are identified from the data.
- For example, use (6) to write:

$$ATE = E\left[\int_0^1 m_1(u,x)du\right] - E\left[\int_0^1 m_0(u,x)du\right].$$

Intuition

- More in general, one can write:

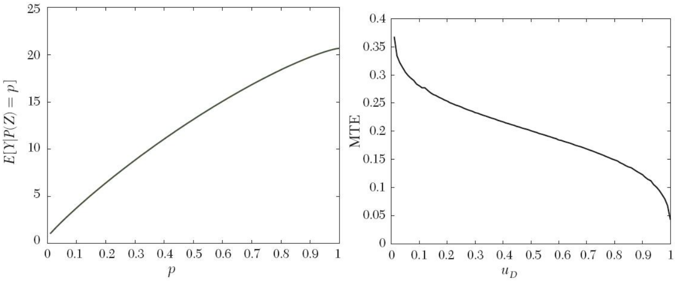
$$
\begin{aligned}
LATE(\underline{u}, \overline{u}) &= E\left[\int_0^1 m_1(u,x)\frac{\mathbb{1}(u \in (\underline{u}, \overline{u}])}{\overline{u}-\underline{u}}du\right] \\
&- E\left[\int_0^1 m_0(u,x)\frac{\mathbb{1}(u \in (\underline{u}, \overline{u}])}{\overline{u}-\underline{u}}du\right],
\end{aligned}
$$

  for compliers at the margin between $P(z_1) \equiv \underline{u}$ and $P(z_2) \equiv \overline{u}$.
- Then knowledge of $LATE(\underline{u}, \overline{u})$ places restrictions on the unknown MTR functions, and hence on the possible values of a target parameter (e.g., ATE, PRTE) other than $LATE(\underline{u}, \overline{u})$.
- One can restrict the MTR functions to lie in some parameter space using parametric or shape restrictions (e.g., additive separability).
- One can then find the set of values for the target parameter that could have been generated by MTR functions that satisfy these restrictions and are also consistent with $LATE(\underline{u}, \overline{u})$.

Tables and Figures

Example of Marginal Treatment Effect

Figure 1: $E[Y|P(Z) = p]$ increases at a diminishing rate in $P = p$, and the MTE decreases in U_D implying diminishing returns to the marginal entrants attracted by increasing values of $P(z)$ (e.g., lowering fees).

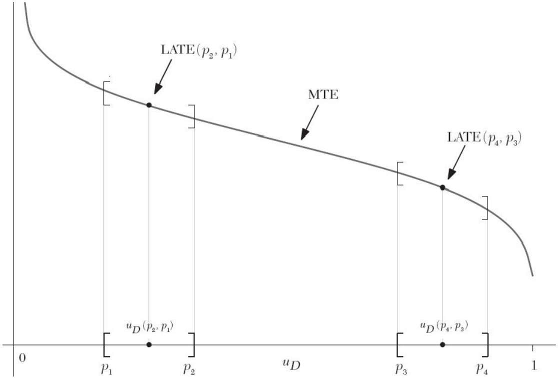


(a) Plot of the $E(Y|P(Z) = p)$

(b) Plot of MTE (u_D): The derivative of $E(Y|P(Z) = p)$ evaluated at points $p = u_D$

Back

Figure 2: The MTE represents the return to schooling for agents indifferent between $D = 1$ and $D = 0$ at all margins of U_D, within the empirical support of $P[Z]$ (conditional on X).



Back

Example: NLSY Data

Figure 3: Estimate $P[Z]$ through a logit model for college participation. The instruments are (jointly) strong predictors of schooling decisions.

TABLE 3—COLLEGE DECISION MODEL: AVERAGE MARGINAL DERIVATIVES

	Average derivative
Controls (**X**)	
Corrected AFQT	0.2826
	(0.0114)***
Mother's years of schooling	0.0441
	(0.0059)***
Number of siblings	−0.0233
	(0.0068)***
Urban residence at 14	0.0340
	(0.0274)
"Permanent" local log earnings at 17	0.1820
	(0.0941)**
"Permanent" state unemployment rate at 17	0.0058
	(0.0165)
Instruments (**Z**)	
Presence of a college at 14	0.0529
	(0.0273)**
Local log earnings at 17	−0.2687
	(0.1008)***
Local unemployment rate at 17	0.0149
(in percent)	(0.0100)
Tuition in 4 year public colleges at 17	−0.0027
(in $100)	(0.0017)*
Test for joint significance of instruments: p-value	0.0001

Back

Example: NLSY Data

Figure 4: Density of $P[Z]$ conditional on $(\tau_1 - \tau_0)X$ (limited support of $P[Z]$ across values of the index). The unconditional support of $P[Z]$ is $[0.0324, 0.9775]$.

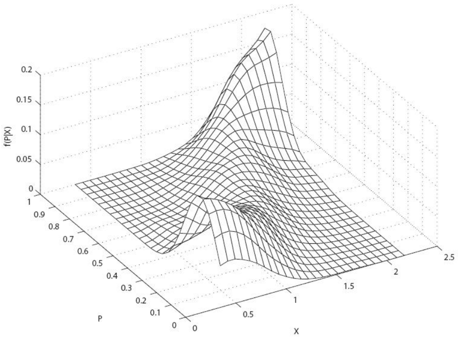


FIGURE 2. SUPPORT OF P CONDITIONAL ON X

Back

Figure 5: The components of X are set at their mean value. The hypothesis of flat MTE is rejected, suggesting self-selection into college based on expected returns.

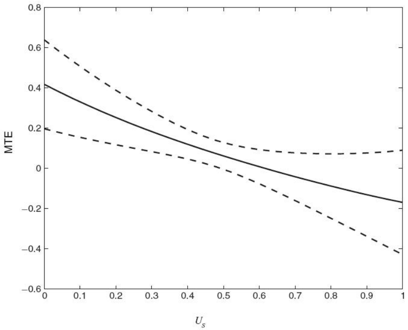


FIGURE 4. $E(Y_1 - Y_0 | \mathrm{X}, U_S)$ WITH 90 PERCENT CONFIDENCE INTERVAL—LOCALLY QUADRATIC REGRESSION ESTIMATES

Back

Example: NLSY Data

Figure 6: In general, the weights are different for IV and MPRTE.

TABLE 1—WEIGHTS FOR MPRTE

Measure of distance for people near the margin	Definition of policy change	Weight
$\lvert \mu_S(\mathbf{Z}) - V \rvert < e$	$Z^k_\alpha = Z^k + \alpha$	$h_{MPRTE}(\mathbf{x}, u_S) = \dfrac{f_{P\mid\mathbf{X}}(u_S) f_{V\mid\mathbf{X}}(F^{-1}_{V\mid\mathbf{X}}(u_S))}{E(f_{V\mid\mathbf{X}}(\mu_S(\mathbf{Z})) \mid \mathbf{X})}$
$\lvert P - U \rvert < e$	$P_\alpha = P + \alpha$	$h_{MPRTE}(\mathbf{x}, u_S) = f_{P\mid\mathbf{X}}(u_S)$
$\lvert \frac{P}{U} - 1 \rvert < e$	$P_\alpha = (1 + \alpha)P$	$h_{MPRTE}(\mathbf{x}, u_S) = \dfrac{u_S f_{P\mid\mathbf{X}}(u_S)}{E(P \mid \mathbf{X})}$

Source: Carneiro, Heckman, and Vytlacil (2010).

Back

Weighted Averages of MTE

Treatment Effects and Estimands as Weighted Averages of the Marginal Treatment Effect

$$\text{ATE}(x) = \int_0^1 \Delta^{\text{MTE}}(x, u_D)\, du_D$$

$$\text{TT}(x) = \int_0^1 \Delta^{\text{MTE}}(x, u_D) h_{\text{TT}}(x, u_D)\, du_D$$

$$\text{LATE}(x, u_D, u_D') = \frac{1}{u_D - u_D'}\left[\int_{u_D'}^{u_D} \Delta^{\text{MTE}}(x, u)\, du\right]$$

$$\text{TUT}(x) = \int_0^1 \Delta^{\text{MTE}}(x, u_D) h_{\text{TUT}}(x, u_D)\, du_D$$

$$\text{PRTE}(x) = \int_0^1 \Delta^{\text{MTE}}(x, u_D) h_{\text{PRTE}}(x, u_D)\, du_D$$

$$\text{IV}(x) = \int_0^1 \Delta^{\text{MTE}}(x, u_D) h_{\text{IV}}(x, u_D)\, du_D$$

$$\text{OLS}(x) = \int_0^1 \Delta^{\text{MTE}}(x, u_D) h_{\text{OLS}}(x, u_D)\, du_D$$

Weights

$$h_{\text{ATE}}(x, u_D) = 1$$

$$h_{\text{TT}}(x, u_D) = \left[\int_{u_D}^1 f(p|X = x)\, dp\right] \frac{1}{E(P|X = x)}$$

$$h_{\text{TUT}}(x, u_D) = \left[\int_0^{u_D} f(p|X = x)\, dp\right] \frac{1}{E((1 - P)|X = x)}$$

$$h_{\text{PRTE}}(x, u_D) = \left[\frac{F_{P^*,X}(u_D|x) - F_{P,X}(u_D|x)}{\Delta\overline{P}(x)}\right], \text{ where } \Delta\bar{P}(x) = E(P|X = x) - E(P^*|X = x)$$

$$h_{\text{IV}}(x, u_D) = \left[\int_{u_D}^1 (p - E(P|X = x)) f(p|X = x)\, dp\right] \frac{1}{\text{Var}(P|X = x)} \text{ for } P(Z) \text{ as an instrument}$$

$$h_{\text{OLS}}(x, u_D) = 1 + \frac{E(U_1|X = x, U_D = u_D) h_1(x, u_D) - E(U_0|X = x, U_D = u_D) h_0(x, u_D)}{\Delta^{\text{MTE}}(x, u_D)}, \quad \text{if } \Delta^{\text{MTE}}(x, u_D) \neq 0,$$

$$= 0 \text{ otherwise}$$

$$h_1(x, u_D) = \left[\int_{u_D}^1 f(p|X = x)\, dp\right]\left[\frac{1}{E(P|X = x)}\right]$$

$$h_0(x, u_D) = \left[\int_0^{u_D} f(p|X = x)\, dp\right] \frac{1}{E((1 - P)|X = x)}$$

Back

Weighted Averages of MTR Functions

Weights for a Variety of Target Parameters

Target Parameter	Expression	Weights: $\omega_0(u, z) \equiv \omega_0(u, x, z_0)$
Average Untreated Outcome	$E[Y_0]$	1
Average Treated Outcome	$E[Y_1]$	0
Average Treatment Effect (ATE)	$E[Y_1 - Y_0]$	-1
Average Treatment Effect (ATE) given $X \in \mathcal{X}^\star$	$E[Y_1 - Y_0 \| X \in \mathcal{X}^\star]$	$-\omega_1^\star(u, z)$
Average Treatment on the Treated (ATT)	$E[Y_1 - Y_0 \| D = 1]$	$-\omega_1^\star(u, z)$
Average Treatment on the Untreated (ATU)	$E[Y_1 - Y_0 \| D = 0]$	$-\omega_1^\star(u, z)$
Marginal Treatment Effect at $\overline{u}$	$E[Y_1 - Y_0 \| U = \overline{u}]$	-1
Local Average Treatment Effect for $U \in (\underline{u}, \overline{u}]$ (LATE$(\underline{u}, \overline{u})$)	$E[Y_1 - Y_0 \| U \in (\underline{u}, \overline{u}]]$	$-\omega_1^\star(u, z)$
Policy Relevant Treatment Effect (PRTE) for new policy $(p^\star, Z^\star)$	$\frac{E[Y^\star] - E[Y]}{E[D^\star] - E[D]}$	$-\omega_1^\star(u, z)$
Additive PRTE with magnitude α	PRTE with $Z^\star = Z$ and $p^\star(z) = p(z) + \alpha$	$-\omega_1^\star(u, z)$
Proportional PRTE with magnitude α	PRTE with $Z^\star = Z$ and $p^\star(z) = (1 + \alpha)p(z)$	$-\omega_1^\star(u, z)$
PRTE for an additive α shift of the jth component of Z	PRTE with $Z^\star = Z + \alpha e_j$ and $p^\star(z) = p(z)$	$-\omega_1^\star(u, z)$
Average Selection Bias	$E[Y_0 \| D = 1] - E[Y_0 \| D = 0]$	$\frac{\mathbb{1}[u \le p(z)]}{P(D=1)} - \frac{\mathbb{1}[u > p(z)]}{P(D=0)}$
Average Selection on the Gain	$E[Y_1 - Y_0 \| D = 1] - E[Y_1 - Y_0 \| D = 0]$	$-\omega_1^\star(u, z)$
Sum of two quantities $\beta_A^\star, \beta_B^\star$ with common measure $\mu^\star$	$\beta_A^\star + \beta_B^\star$	$\omega_{A,0}^\star(u, z) + \omega_{B,0}^\star(u, z)$